Radiation efficiency of acoustic guitars

J. C. S. Lai and M. A. Burgess
Department of Mechanical Engineering, University College, The University of New South Wales, Canberra, ACT 2600, Australia

(Received 31 October 1988; revised 3 April 1990; accepted 7 May 1990)

The transfer functions between acceleration and force ("accelerance") at the driving point and radiation efficiencies for the top plates of guitars have been investigated. A vibration exciter was used for driving the guitar at the bridge and the transfer functions and cross-power spectra were measured at the driving point with the aid of an impedance head. The frequency spectrum of the accelerance function determined in this manner showed good agreement with those reported for the same guitar using a different excitation and measurement method. The output acoustic intensity was measured with a sound-intensity probe and the radiated sound power from the top plate was determined. The radiation efficiency, determined from the ratio of the top plate radiated sound power and the input power, is presented for different guitars and for a guitar with a modified top plate.

PACS numbers: 43.75.Gh

INTRODUCTION

Considerable research has been undertaken on the vibration properties of stringed musical instruments, in particular violins and to a lesser extent guitars (e.g., Chicago papers, 1982). Most of this work has involved studies of the vibration properties of the plates themselves. A visual representation of the nodal lines can be achieved by observation of the movement of sawdust or by holographic techniques. The latter technique also gives data on the amplitude as well as the modal geometry. Input impedance measurements across the top plate of a guitar were made by Firth (1977) and an analogous acoustical circuit was presented to describe the action of the guitar in the range of the air resonance and the first top-plate resonance. An attempt was made by Caldersmith (1978) to model the guitar as a reflex enclosure to describe its low frequency behavior. Frequency response curves can be obtained with the aid of accelerometers (e.g., Caldersmith, 1986).

The traditional methods used by the violin makers for assessment of the instrument involve tapping the plate and listening to the radiated sound. There have been a limited number of investigations utilizing measurements of the actual radiated sound from the instruments. For one study by Tro *et al.* (1983), a large array of microphones was used to determine the intensity vectors from a double bass.

With modern measuring instrumentation, such as sound intensity probes, fast Fourier transform analyzers and computers, it is now feasible to determine the sound power radiated from a sound source. The input power can be determined, the radiation efficiency calculated and comparisons between different instruments made. It also enables monitoring of the changes to radiation efficiency following changes to the design or characteristics of an instrument.

Suzuki (1986) has reported on measurements of the vibration and sound radiation of a piano soundboard excited via a broadband noise signal from a small vibration exciter at a fixed driving point. The radiated sound intensity was determined from measurements with an accelerometer and a pressure microphone at a number of points over the surface of the piano soundboard; the radiation efficiency for the frequency range of interest was then determined.

This paper presents the results of measurements on the top plates of guitars using a similar approach to that taken by Suzuki (1986). The major differences were that the measurements were made on complete instruments and the sound intensity was measured with a sound intensity probe.

I. MEASUREMENTS AND ANALYSIS

A schematic diagram of the instrumentation set up is shown in Fig. 1.

A. Guitar

The tension on the strings of the guitar to be tested was first checked to ensure that the instrument was "in tune." Wedges of foam were then placed at two points along the neck, one close to the body of the instrument and the second two thirds of the way along the neck (see Fig. 2). The purpose of the foam was to dampen the vibration of the strings during the testing.

The head of the guitar was tied to a metal framework. The suspension allowed for movement of the guitar only in

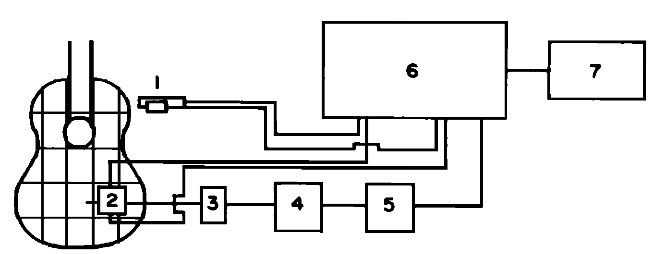

FIG. 1. Schematic diagram showing the measurement arrangements for determination of the input and output power for a guitar. The components are: 1, sound intensity probe; 2, impedance head; 3, vibration exciter; 4, amplifier; 5, filter; 6, FFT analyzer; and 7, computer.

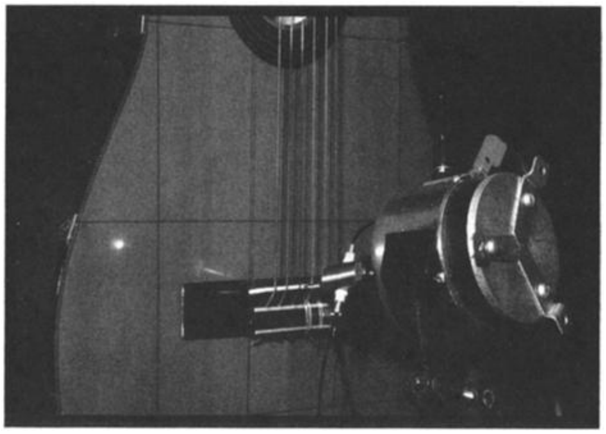

FIG. 2. Photograph of a guitar with the vibration exciter and impedance head attached via the small wire.

the plane normal to the top plate. The experiment was conducted in a room with a volume of about 85 m^3 and a reverberation time of about 0.8 s. Even though there would be negligible effect on the acoustic intensity measurements, a screen of fabric covered fiberglass was placed behind the guitar (but not touching) to minimize unwanted reflections of sound radiated from the rear of the guitar.

Two guitars were tested. The first was a standard, commercially-available guitar, hereinafter referred to as guitar A. The second was a guitar, hand-made to standard specifications, hereinafter referred to as guitar B. This guitar had been used for extensive vibration tests by the maker using a different driving system. The results of these previous tests enabled comparisons to be made with the results obtained using the test methods reported in this paper.

B. Excitation

The excitation was provided by a small vibration exciter that was supported horizontally. The exciter was driven by a broadband noise signal generated with a Bruel and Kjaer FFT analyzer, type 2032. The frequency range for the broadband noise corresponded to the frequency range selected for the measurements, typically 800 Hz. The signal was amplified using a Bruel and Kjaer power amplifier, type 2706, and, for some of the tests, a Bruel and Kjaer tunable bandpass filter, type 1621, was used to restrict the driving frequency range to within a 3% frequency band.

A Bruel and Kjaer impedance head, type 8001, was attached to the exciter with an extension rod, approx, 130 mm long. The impedance head was comprised of an integrated piezoelectric accelerometer and force gauge mounted very close to the driving point. A number of different methods for connecting the exciter unit to the guitar were investigated. The most reliable, repeatable and reproducible method was found to be that using a short wire, 14 mm long, fixed at the end of the impedance head. This wire was pushed firmly into a small hole that had been drilled in the saddle (i.e., on the bridge) of the guitar midway between the B and E strings.

The signal to the vibration exciter was maintained at as high a level as possible. This was observed as being the point at which further increases in the input level would lead to bouncing of the contact wire.

C. Input power

The input power was determined from measurement of the cross-power spectrum between the force and the acceleration transducers in the impedance head, i.e., at the driving point. The signals from these transducers were transmitted via associated charge amplifiers to the two channels of the FFT analyzer. This is an 800 line analyzer. Since the lower frequency range was of most interest, for the majority of the measurements a frequency range of 800 Hz was used, thus providing a resolution of 1 Hz per line. Spectra were obtained using a Hanning window with 50% overlap. The frequency data for the cross-power spectra, each one based on 100 averages, were stored in a dedicated Hewlett–Packard series 300 micro-computer. The input power was subsequently calculated from this data using the formulation shown in the Appendix. The accelerance function, by which we mean the transfer function between acceleration and force at the driving point, was obtained at the same time as the cross-power spectrum, using the same input signals to the FFT analyzer.

D. Output sound power

The sound power radiated from the top plate of a guitar was determined by the sound intensity technique. The technique for measuring sound intensity (that is, sound power flux density) with two pressure microphones and its limitations have been discussed in great detail in the literture [see, for example, Fahy (1989)] and will not be repeated here. The face of the guitar was marked into 25 segments as shown on Fig. 1. A Bruel and Kjaer sound intensity probe, type 3519, fitted with two 12.7-mm microphones was scanned slowly over each segment while 100 averages were obtained by the analyzer using a Hanning window with 50% overlap. The distance from the end of the probe to the face of the guitar was approximately 10 mm. For most of the measurements, a 12-mm spacer between the two microphones was used. As the recommended lower frequency limit for the probe fitted with this spacer is of the order of 125 Hz, check measurements were made with a 50-mm spacer between the two microphones and no significant differences in the results were observed. The 12-mm spacer was preferred because of the practical problems associated with the physical dimensions of the probe with the larger spacer.

The sound intensity spectrum, in terms of W/m^2, for each of the segments was stored in the computer for subsequent analysis.

E. Analysis

For each test, the total sound power output from the top plate for each frequency was determined by multiplying the sound intensity for each segment by the area of that segment. The input sound power for each frequency was determined from the average of at least three samples. The radiation efficiency for each frequency was determined from the ratio

of the sound power radiated from the top plate to the input power.

For various other investigations, the overall input and output powers for each frequency were determined. Plots of the input cross-power spectra, accelerance spectra, output sound intensity, etc., were produced from the stored data over the frequency range of interest.

II. RESULTS AND DISCUSSION

A. Accelerance spectra and sound intensity contours

The accelerance function was determined from the ratio of acceleration to the input force at the driving point. Hence, the accelerance spectrum can be used to indicate the frequencies of the natural modes for the top plate of the guitar. A comparison between the accelerance spectrum determined using the equipment described above with that determined by Caldersmith (1986) for guitar B is shown in Fig. 3. Note that the accelerance curve obtained in this study has been shifted upward in Fig. 3 to allow for comparison with Caldersmith's results (1986). For his tests, Caldersmith (1986) used a sinusoidal source with constant force via the driving coil of a small loudspeaker at approximately the same point on the bridge. The signal from an accelerometer, attached to the bridge just behind the driver, then represented the accelerance spectrum. The agreement between the accelerance spectra obtained by the two different methods is extremely good. The greatest variation occurs around 500 Hz where it is possible that the logarithmic frequency scale used by Caldersmith (1986) could not resolve the two peaks shown on the narrow-band analysis using the FFT analyzer.

Caldersmith (1986) has described the monopole and multipole modes of the guitar and identified these modal frequencies for guitar B. The fundamental (0,0 mode) resonates twice because of the coupling between the plate and the enclosed volume of air via the sound hole. The lowest resonance, which is at the Helmholtz air resonance, occurs when the top plate and air piston move out of phase and corresponds to the peak in the accelerance spectrum around 91 Hz. The upper resonance (which is also referred to as the first top plate resonance in the literature) occurs around 189

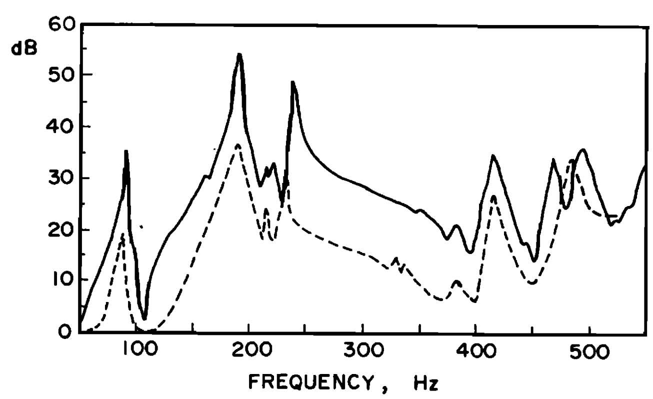

FIG. 3. Comparison of the accelerance spectrum for guitar B determined using two different methods; —as described in this paper, - - -as described by Caldersmith (1986).

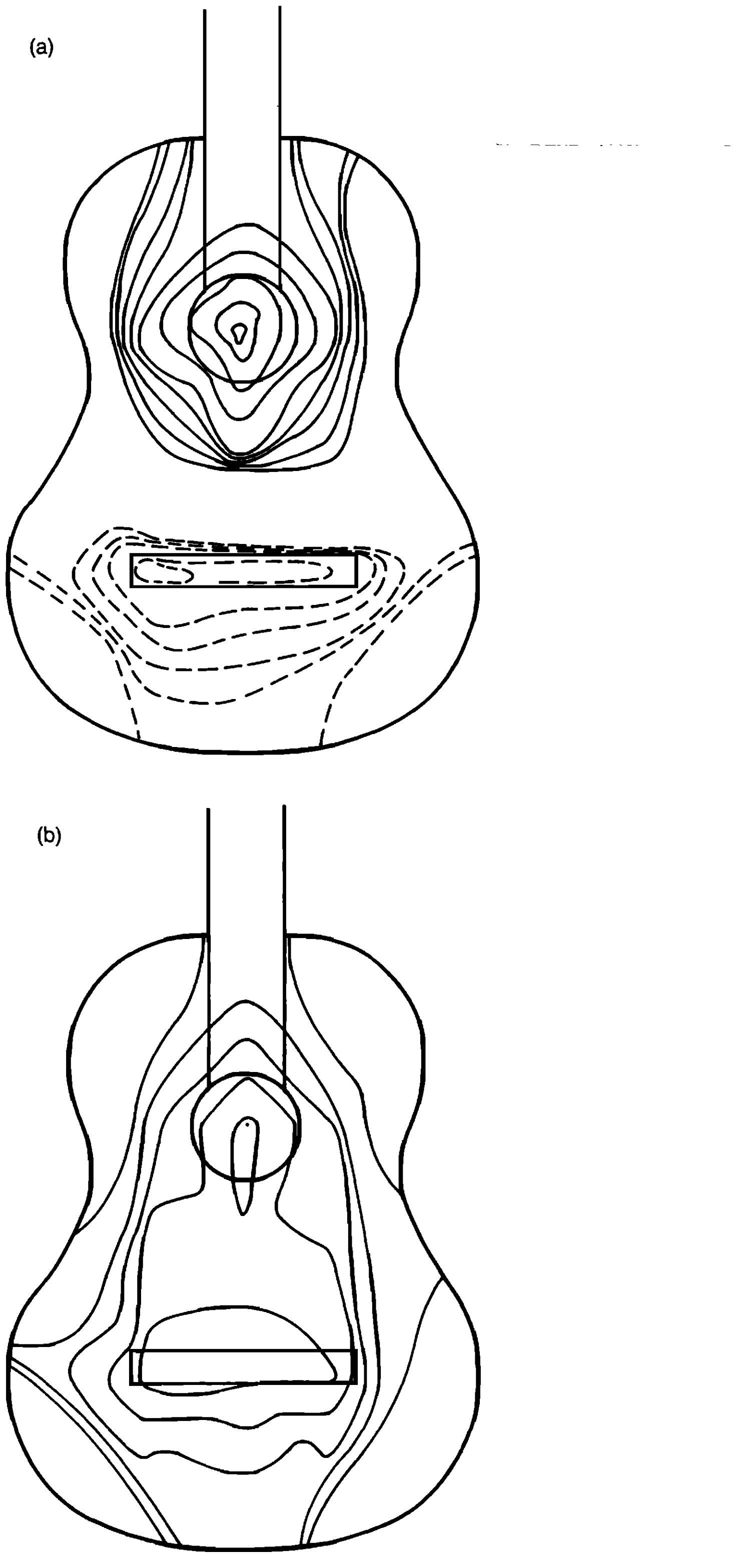

FIG. 4. Sound intensity contours for guitar B. (a) At 91 Hz: —, positive contour, 71–87 dB in steps of 2 dB; - - -, negative contour, − 70 to − 75 dB in steps of 1 dB. (b) At 189 Hz: —positive contour, 61 to 69 dB in steps of 2 dB.

Hz, when the air piston and the guitar top plate resonate in phase. As shown in Fig. 4(a) and (b), the sound intensity contours on the top plate of the guitar support the above interpretation for the air resonance mode and the upper reso-

nance mode respectively. For the Helmholtz air resonance mode, the sound intensity contours clearly show that the radiation through the sound hole dominates.

The relationship between the resonance of the front and back plates can then lead to two modes; one, the 0,0-0,0 mode, where the front and back plates move in antiphase so that there is maximum change in the enclosed volume of air. For the second, the 0,0 + 0,0 mode, the two plates move in cophase. The 0,0-0,0 mode corresponds to the large peak at 189 Hz in Fig. 3. The 0,0 + 0,0 mode corresponds to the small peak around 215 Hz which, for the test method described in this paper, appears as a pair of closely coupled modes.

Caldersmith (1986) also described the cross dipole mode (1,0), where the nodal axis is along the length of the face plate, which corresponds to the next large peak, around 240 Hz, as shown in Fig. 3. For the long dipole mode (0,1), which normally occurs around 300 to 350 Hz, the nodal axis is in line with the bridge. The tripole mode (2,0) corresponds to the peak around 415 Hz in Fig. 3. This mode is generally considered to be an important mode because it is strongly excited by all the strings and because the two outer anitnodal poles move in antiphase to the central pole, thus acting as an efficient net volume pump. Above this frequency, the peaks in the response curve relate to the higher multipole modes of the guitar that are difficult to identify clearly due to their close proximity to each other. Unfortunately, the experimental grid used has been too coarse to illustrate the dipole and tripole modes clearly through sound intensity contours.

The accelerance spectra for each of the guitars were repeatable under different input powers. The mass of the top plate of a guitar is of the order of 200 g and, with such a light weight, it is possible that the plate could be "mass loaded" by the impedance head, thus influencing the performance of the vibrating plate. The specifications for the impedance head indicate that, while the head itself weighs 29 g, it is only the mass (i.e., 1.3 g) below the force gauge, which can be considered as the load.

Any effect from the impedance head was checked by comparing the accelerance spectrum determined using the vibration exciter and impedance head with the same function found using an impact hammer (Bruel and Kjaer, type 8202). The hammer impact was applied at the same position on the saddle as for the vibration exciter and a small accelerometer was attached to the bridge close to this point. The comparison between the accelerance spectrum obtained for guitar A using the two excitation methods is shown on Fig. 5. The close agreement indicates that the mass loading introduced by the impedance head onto the top plate was negligible.

The effect of gross changes to the mass of the top plate of guitar A was demonstrated by tests for which eight 15-g weights were distributed over the face below the waist, thus increasing the mass from around 200 g to around 320 g. The effect on the accelerance spectrum can be seen on Fig. 6. As expected, there has been negligible effect at the frequencies around the Helmholtz air resonance mode, which is primarily determined by the geometry of the sound box. For the top

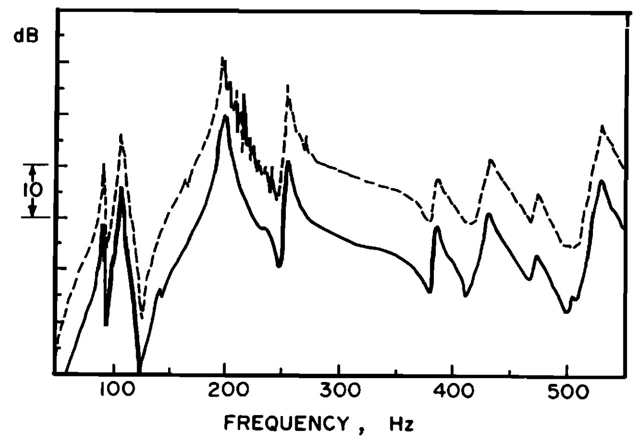

FIG. 5. Comparison of the accelerance spectrum for guitar A determined using two different excitation methods: —, broadband noise applied with vibration exciter; - - -, impact hammer used for excitation.

plate modes, the additional mass has lowered the resonant frequency, with greater differences occurring in the regions corresponding to the dipole and tripole modes.

A comparison between the accelerance spectrum for guitars A and B is shown on Fig. 7 where the differences between the two guitars are clearly evident. The frequencies for the resonant modes for guitar A were approximately 10 to 20 Hz higher than for guitar B. The double peak around 100 Hz for guitar A (that is, the air mode region) results from coupling of the guitar body and the neck. An applied load to the neck of the guitar significantly reduced the magnitude of the lower of the two peaks.

B. Input power

At each frequency, the input power for the experimental arrangement was calculated from the cross-power spectra at the excitation point using the formulation shown in the Appendix. The total input power over the range 50–550 Hz was in the range 10–20 mW. This was the maximum input power that could be used without producing excessive vibrations of the top plate. A comparison of the input power spectra for

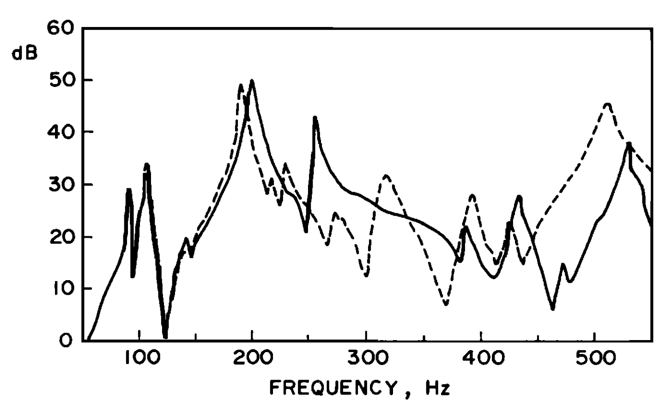

FIG. 6. Comparison of the accelerance spectrum for guitar A under two different conditions: —, as normal; - - -, with additional mass distributed over the top plate.

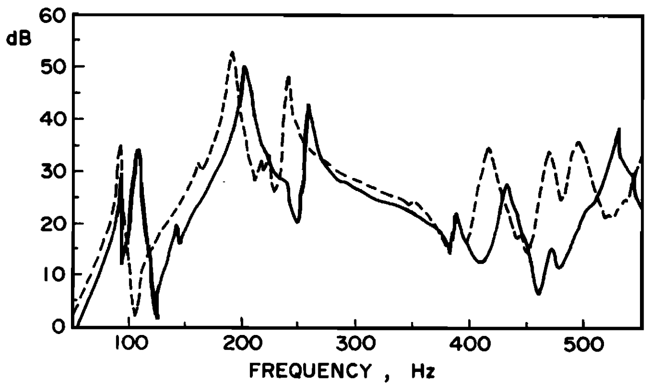

FIG. 7. Comparison of the accelerance spectrum for two different guitars: —, guitar A; - - -, guitar B.

guitars A and B, normalized with respect to the total input power over the frequency range, is shown in Fig. 8.

The frequency distribution of the input power is somewhat different from the shape of the accelerance because maximum power transfer from the shaker to the guitar occurs under conjugate matching conditions (i.e., when the real parts of the impedances are equal and the imaginary parts are equal in magnitude but opposite in sign).

The very low input power within the frequency range of interest caused problems with the determination of the radiation efficiency. At these frequencies, high values of radiation efficiency were obtained and since the output sound power is divided by such a very small number, these high radiation efficiency values cannot be accepted with confidence, considering the limits of experimental error. In an attempt to overcome the low input power over some of the frequency range, the broadband excitation signal was filtered to concentrate the input energy over a narrow frequency range. A 3% bandpass filter was centered at 300 Hz, and it was still not possible to provide sufficient input power to the plate for reliable radiation efficiencies to be obtained. Even though the bandpass filter was sharply defined, there was some energy at the other frequencies due to the nonperfect cutoff of the filter. For the resonance peaks around 150 and 400 Hz, there was sufficient input power to obtain radiation efficiencies comparable with those obtained when the broadband signal was used. Sinusoidal excitations at around 180 and 300 Hz were used to check the measurements made with broadband excitations and the results agreed to within 5%.

C. Radiation efficiency

The overall radiation efficiency (that is, the ratio of top plate radiated sound power to the input power) over the frequency range of interest, namely 50 to 550 Hz, was 0.14 for guitar A and 0.12 for guitar B. The repeatability of these values with different levels of input power was within 0.01. When the top plate of guitar A was loaded with additional mass, the efficiency reduced to 0.08.

As mentioned in Sec. II B, the input power was very low over portions of the frequency range. This could lead to very high values of radiation efficiency at these frequencies if only a very small output sound power was measured. For the investigations on the guitars, the limits of measurement accuracy corresponded to an input power of the order of 10^{-3}–10^{-4} mW. When the input power was less than this value, the radiation efficiency was often close to, or exceeding, 1. For the frequency analysis, the data for the radiation efficiency was rejected if the input power was less than 10^{-3} mW, as the value obtained was considered to be unreliable. As mentioned in Sec. II B, results obtained with sinusoidal excitations at around 180 and 300 Hz agree with those obtained with broadband excitations and hence support the validity of the approach adopted.

A comparison between the radiation efficiencies for guitars A and B, with similar input powers, is shown in Fig. 9. The radiation efficiencies show similar trends for the two guitars with both showing high radiation efficiencies in the 400-Hz region, where the tripole mode is expected. From the accelerance spectrum for the two guitars, as shown on Fig. 7, the frequencies for the various modes were slightly lower for guitar B than for guitar A. A similar shift towards the lower frequencies for the peaks of the radiation efficiencies for guitar B can also be seen.

The radiation efficiencies for guitar A with the top plate loaded with additional mass can be seen in Fig. 10. The response in the low frequencies (that is, the Helmholtz air resonance region) shows minimal change because the Helmholtz air resonance is primarily determined by the geometry

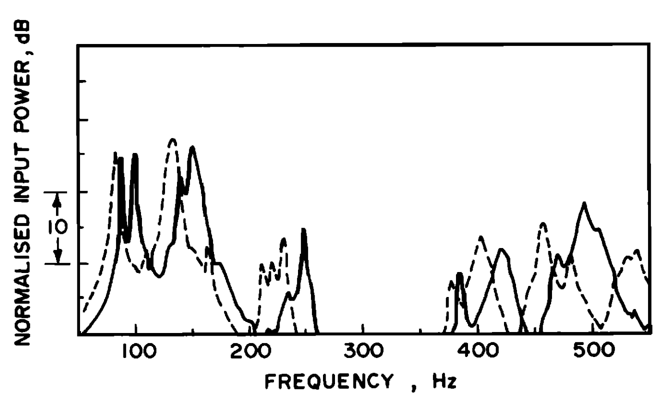

FIG. 8. Comparison of the normalized input power spectra for two different guitars: —, guitar A; - - -, guitar B.

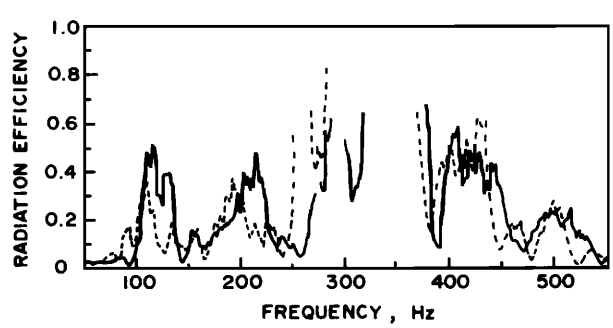

FIG. 9. Comparison of the radiation efficiency spectra for two different guitars: —, guitar A; - - -, guitar B.

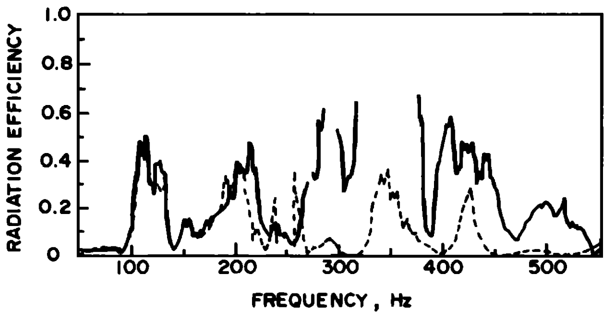

FIG. 10. Comparison of the radiation efficiency spectra for guitar A under two different conditions: —, as normal; - - -, with additional mass distributed over the face.

of the sound box and, as shown in Fig. 4, most of the sound energy is radiated from the vicinity of the sound hole. At higher frequencies, such as in the region of the tripole mode, around 400 Hz, there has been a significant reduction in the radiation efficiency because the radiation through the sound hole is not the dominant factor and the additional mass increases the impedance of the top plate.

III. CONCLUSIONS

Based on similar work on a piano sounding board by Suzuki (1986), a technique for investigation of the accelerance spectrum and the radiation efficiency of the top plates of guitars has been developed.

The method involved driving the guitar with a vibration exciter using broadband excitation and measuring the transfer function and the cross-power spectrum at the driving point. The former was used to identify the modes and the latter for determination of the input power. The output acoustic intensity was measured with an intensity probe and the sound power determined. The radiation efficiency was then determined from the ratio of the top plate radiated sound power to the input power.

Comparison of the accelerance spectrum determined using this method showed good agreement with those obtained for the same guitar using different transducers and excitation methods. The input power spectrum was not uniform across the frequency range because of the resonant characteristics of the guitar. For those frequencies where the input power was extremely small, it was not possible to obtain reliable values for the radiation efficiencies. At other frequencies, the radiation efficiencies were found to be repeatable for each guitar and differences were observed between different guitars. The importance of the cross tripole mode, long considered to be one of the most important modes for the sound radiation of guitars, was confirmed. Alterations to the guitar face led to changes in the radiation efficiency at the higher modes but, as would be expected, not in the region of the Helmholtz air resonance mode, where it is the radiation via the sound hole that is important and not the vibration of the plate.

The methods that have been developed can be used to assess the changes to the radiation efficiency of guitars when alterations are made. This is more useful than observations of the accelerance spectrum, which only gives an indication of the changes in the modal frequencies but not the efficiency with which those modes are radiated. The methods can also be adapted for investigations of the sound from other radiating bodies.

ACKNOWLEDGMENTS

Without the valuable advice, guidance, and support of Graham Caldersmith and Neville Fletcher, this work would not have been possible. The assistance of Andrew Dombek with the measurements is gratefully acknowledged.

APPENDIX: FORMULATION OF INPUT POWER FROM THE CROSS-POWER SPECTRA OF THE FORCE AND ACCELERATION TRANSDUCERS

The power spectral density $P(\omega)$ is related to the force spectrum $F(\omega)$, and the velocity spectrum, $V(\omega)$ by the relationship

$$P(\omega) = \mathrm{Re}[F^*(\omega)V(\omega)], \tag{A1}$$

where Re denotes the real part and F^* is the complex conjugate of F. Since the velocity spectrum $V(\omega)$ and acceleration spectrum $A(\omega)$ are related by

$$V(\omega) = \frac{1}{j\omega}A(\omega). \tag{A2}$$

From Eqs. (A1) and (A2), the power spectral density $P(\omega)$ is related to the imaginary part (Im) of the cross-power spectrum between F and A by

$$P(\omega) = \omega^{-1}\,\mathrm{Im}[F^*(\omega)A(\omega)]. \tag{A3}$$

Hence, the input power P for the frequency range $\omega_i \leqslant \omega \leqslant \omega_f$ can be calculated from

$$P = \mathrm{Im}\int_{\omega_i}^{\omega_f} \frac{1}{\omega}\,[F^*(\omega)A(\omega)]\,d\omega. \tag{A4}$$

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